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Spin models as microfoundation of macroscopic market models

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HIGHLIGHTS

- We demonstrate the microfoundation of macroscopic financial market models.
- We start with an agent-based model closely related to the Ising model.
- We deduce a macroscopic Langevin equation close to common price evolution models.
- With combined micro- and macro-description we improve the model understanding.

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ABSTRACT

Macroscopic price evolution models are commonly used for investment strategies. There are first promising achievements in defining microscopic agent based models for the same purpose. Microscopic models allow a deeper understanding of mechanisms in the market than the purely phenomenological macroscopic models, and thus bear the chance for better models for market regulation. However microscopic models and macroscopic models are commonly studied separately. Here, we exemplify a unified view of a microscopic and a macroscopic market model in a case study, deducing a macroscopic Langevin equation from a microscopic spin market model closely related to the Ising model. The interplay of the microscopic and the macroscopic view allows for a better understanding and adjustment of the microscopic model, as well, and may guide the construction of agent based market models as basis of macroscopic models.

1. Introduction

While extensively used textbook models of financial markets assume an efficient market [1] and Gaussian distributed returns of financial assets [2,3], real markets show a completely different behavior with huge bubbles, crashes, and erratic asset price time series. This discrepancy is believed to be one of the central reasons for the 2008 financial market crisis [4]. Scientific investigations beyond the standard theories of financial markets are partly driven by methods from theoretical physics, especially in the description and modeling of macroeconomic quantities (e.g. price time series), as well as in the creation of microeconomic models with many interacting agents [5].

Price time series of markets exhibit universal properties, known as stylized facts, that do not occur in equilibrium models of financial markets: Returns are power law distributed [6] and their signs are uncorrelated, volatility is clustered in time [7], and multi-fractal properties occur [8]. The first three stylized facts have been incorporated in different volatility forecasting models (broadly used for investment strategies) like ARMA and GARCH variants (for an overview and comparison see Ref. [9]). A recent development also includes multi-fractal properties with reasonable success [10,11]. Microeconomic studies have led to a number of models (see Ref. [12] for an overview) which, however, are mostly in the stage of anecdotal

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models. A prominent early agent model for herding is the Kirman ants herding model [13]. In an analytic description it gave rise to a particularly simple class of herding models that generate the typical stylized facts of financial time series [14,15]. Another early model, the Lux Marchesi model [16], introduces chartist agents competing with fundamentalists, leading to power law distributed returns as observed in real markets, contradicting the popular efficient market hypothesis. A minimalistic spin model [17] introduces spatial structure and exhibits pronounced bull and bear markets with a long memory as in real markets [18].

As proposed by Farmer [5] and exhibited in Refs. [19,20], agent based models can be brought so close to real markets that they are candidates for investment strategy tools. While in Refs. [19,20] the outcome of agent based models was directly compared to price time series, we here deduce a macroscopic price equation from the microscopic market model of Ref. [17] which is based on a modified Ising spin model. Macro-relations for agent based models are discussed in Refs. [21–23] and suggested for the spin model [17] in Ref. [24]. There are several advantages of this procedure: the focus shift from macro to micro-models is facilitated, and the interplay of microscopic and macroscopic views assists agent based model construction.

We here focus on the previously published spin model [17], since it allows for a clear geometric insight, connects to the well known Ising model, and has been studied as a minimalistic agent based model exhibiting stylized facts [18,25]. We first analyze the dynamics of the model with a frozen coupling to the global magnetization of the system and find a phase transition between small and large magnetizations with a jump in volatility. Similar phase transitions are found in other socio-economic models, as well [26]. Then we construct a macroscopic equation for the price, which includes a volatility switching due to the phase transition, reminiscent of regime switching models [9]. The macroscopic model is compared to the microscopic one with good agreement and thus provides insight in the mechanism ruling the spin market model. From the macroscopic point of view we investigate the role of the model parameters leading to instructions for the adjustment of the feedback for a larger system. Consequently, we propose the use of deduced macro-equations as a guiding principle for the construction of agent based models.

2. The model

The spin model of Ref. [17] describes $N = L^2$ agents on a square lattice with periodic boundary conditions. The agents can decide between two states $S_i = \pm 1$ (buy or sell), according to two conflicting forces: On the one hand they tend to do what their neighbors do (as in the Ising model), while on the other hand they tend to panic-like behavior with increased switching probabilities, if minority and majority sizes diverge. This is implemented by a local field

$$h_i = \sum_{j\,\in\,\mathrm{nn}(i)} S_j - |h| \cdot S_i \tag{1}$$

$$h = \alpha \cdot m = \alpha \cdot \frac{1}{N} \sum_{k=1}^{N} S_k \tag{2}$$

with $\mathrm{nn}(i)$ denoting the nearest neighbors of agent i. In random sequential update, a randomly chosen agent i evolves according to the local field h_i. Its new state S_i is determined by the probabilities

$$p(S_i = \pm 1) = \frac{1}{1 + \exp(\mp 2\beta h_i)}. \tag{3}$$

For $\alpha = 0$ this corresponds to a simulation of the pure Ising model with a heat bath algorithm and coupling $J = 1$. If $\alpha \neq 0$, the system couples to its mean field h as a time-dependent global quantity (comparable to the well known global property "price" in economy). Note that this coupling is of an entirely different type than the coupling of a spin to an external magnetic field, the latter being described by a term $-h$ instead of $-|h| \cdot S_i$. In Ref. [17] the term $-|h| \cdot S_i$ was motivated with the minority game [27], as it increases the flip probability, and the probability to join the minority with a single flip is proportional to the majority size. However, combined with the interaction with neighbors, the effect of this term is rather to induce panic-like behavior with many consecutive flips. As shown in Ref. [28, see especially Appendix A therein], this can be seen as an increase of the market temperature. The addition of the panic term results in a much more complex behavior compared to the Ising model. The time dependence of h introduces a feedback to the system's dynamics. Later on we will fix h to certain constant values in order to understand the role of $h(t)$.

The dynamics of this model exhibits large and broadly distributed fluctuations in the magnetization m that we here relate to fluctuations in financial markets. For this purpose, price has been introduced via additional fundamentalists, leading to an expression for logarithmic returns, which is identified with the change of magnetization in the model, $\mathrm{ret}(t) = \ln(p(t+1)/p(t)) \propto \Delta m$ [18] (sweeps are considered as time steps). As can be seen in Fig. 4 on the left, where the absolute return distribution of the model is shown with the black line, the returns are distributed with a truncated power law and an exponent of approximately 3 (or exponent 2 for the cumulative distribution). In addition, a long-range autocorrelation (with typical timescales in contrast to real markets) is reported [18].

In order to characterize the lattice state, the border line length

$$l_\mathrm{b} = \sum_{\langle ij \rangle} \Theta(-S_i S_j) \tag{4}$$

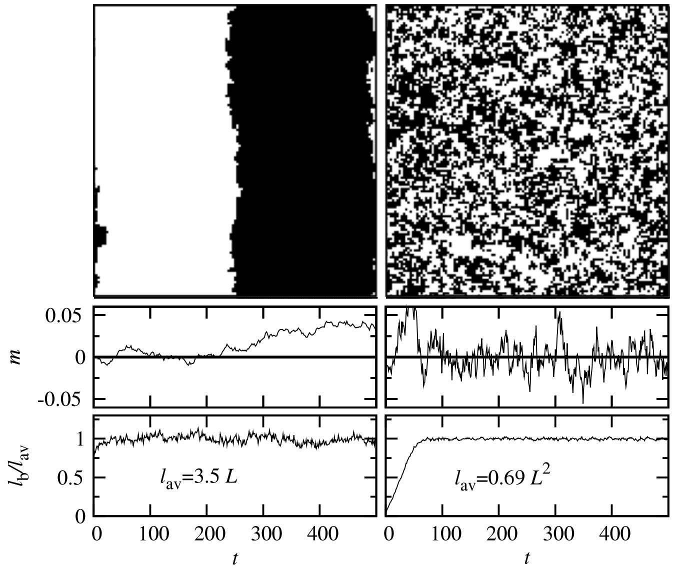

Fig. 1. From top to bottom: lattice states ($L = 128$), magnetization m, and borderline length time series for $h = 1.5$ (left column) and $h = 2.0$ (right column).

is introduced, where $\langle ij \rangle$ are pairs of nearest neighbors and $\Theta(1) = 1,\ \Theta(-1) = 0$. This property is related to the number of agents, which are exposed to stress due to different neighbors and hence switch more often (The other agents switch with a probability close to zero. On a side note, this allows for a faster algorithm). Most of the time l_b is near $2L$ (compare Fig. 3) indicating that the system is in striped states (compare Fig. 1, upper left). Such striped structures are reported for opinion formation models with ordering dynamics [29], where they occur in finite systems in the coarsening process as metastable states of long lifetimes.

3. Frozen feedback

To simplify the dynamics, let us now set the mean field h in Eq. (1) to a constant value (i.e. treat it as an external constant field). Because the dynamics of the spin model favors striped states, we use $S_i = 1$ for $x < \frac{L}{2}$ and $S_i = -1$ for $x \geq \frac{L}{2}$ as the initial state. Fig. 1 shows the state of the lattice (for the fixed time $t = 400$ sufficient to guarantee a typical shape of the grid states) and the time-dependent magnetization and borderline length for a small (left column) and a large h (right column). In this paper, the temperature is set to $0.2\ T_c$ with T_c being the critical temperature of the Ising model.

In the case with a small constant value of h (typical in this model), the borders diffuse leading to a typical border shape (with almost constant l_b) and varying magnetization. Because one of the regions finally will vanish, these states are metastable for frozen h, but with large lifetimes [29]. The returns Δm are Gaussian distributed and not considerably correlated (the same holds for higher moments of Δm). Therefore we conclude that the magnetization performs a random walk. This can be modeled by the macroscopic equation

$$\Delta m(t) = \sigma(L, h) \cdot \xi_t \qquad (5)$$

with the normally distributed and independent random variable ξ_t.

For larger h, we get a completely different behavior, indicating a phase transition. The system is disordered globally and l_b is much larger (scaling with L^2 instead of L). Δm is much larger, as well, and a strong drift leads to fast fluctuations of m around zero. Fig. 2 shows the standard deviation σ as a function of h for different L, calculated as the average over 10 000 time steps after typical states are reached. To conserve the metastable striped states, all agents at $x = L/4$ are fixed to $S_i = 1$, at $x = 3L/4$ to $S_i = -1$. On the left, the data is multiplied with $(L/128)^{3/2}$. For metastable striped states (small h), then the curves collapse for different L indicating a scaling with $L^{-3/2}$. On the right hand side, the disordered part of the curves collapses with a scaling $\propto L^{-1}$. This is connected to a phase transition near $h_{crit} = 1.84$. Due to the low bulk noise for this system, it is close to the Generalized Voter Model (GVM) phase transition with a jump in the order parameter [30,31].

The scaling properties can be understood as follows: The number of active agents (those with different neighbors) is proportional to l_b. Therefore, the number of steps contributing to a single return is proportional to l_b, and thus the total change expressed in steps scales with $l_b^{1/2}$, if the single steps are treated as independent. One could say, the "speed of time" changes with l_b which is reminiscent of the time transforming procedure in multi-fractal models [10,11]. Including the pre-factor of $1/L^2$ in the magnetization and with the scaling properties of l_b (compare Fig. 1) the observed scaling is found.

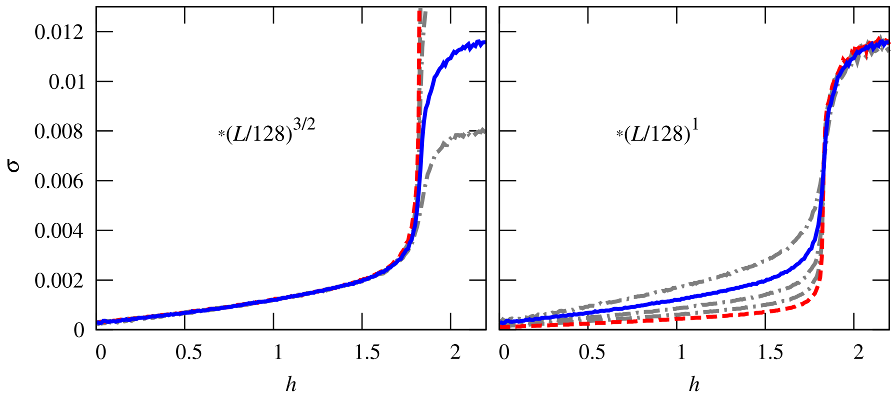

Fig. 2. Standard deviation σ for fixed values of the field h below and above the phase transition. $L = 128$ (blue solid line), $L = 1024$ (red dashed) and $L = 64, 256, 512$ (gray dash-dotted). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

4. Macroscopic equation

Since in the spin market model the system is in ordered states most of the time, we can use a generalization of Eq. (5) with a smoothened field $\bar{h}(t) = \alpha\bar{m}(t)$,

$$\Delta m(t) = \sigma\,(L, \alpha\bar{m}(t)) \cdot \xi_t. \tag{6}$$

This equation describes the price evolution as a macroscopic quantity. The dependence $\sigma(L, h)$ is taken from fits to the curves in Fig. 2. Eq. (6) is valid only for fields $\bar{h} = \alpha\bar{m}$ beyond the phase transition, because in the disordered phase drift-forces and geometric effects have to be considered and the coarsening process is not taken into account. We can put it as follows: The system's magnetization performs a random walk described by Eq. (6) and additionally experiences the effect of complex and finally reflecting borders around $m_{\text{crit}} = h_{\text{crit}}/\alpha$ as can be seen in Fig. 3 on top.

Since here we will not describe the action of the borders at m_{crit}, we will use Eq. (6) for supercritical m too and use the time series $h(t)$ produced by the microscopic model for the further investigation of the macroscopic model. The bottom panel of Fig. 3 shows the comparison of the quantities $\bar{\sigma}(t)$ as the volatility of the microscopic model and $\sigma(L, \bar{h}(t))$ as the according property assigned to the macroscopic Eq. (6). In this paper, all averages are performed over 30 time steps. The reasonable agreement allows for the conclusion that the system can be understood through typical diffusing borders according to the actual $\bar{h}$. The use of Eq. (6) is justified.

Before we continue by using Eq. (6) to achieve a deeper understanding of the microscopic system, we reformulate the process to arrange our findings in order of price evolution models commonly used for investment strategies [9]. Many price evolution models (in the detrended version with discrete time, we skip details concerning the discretization) can be written as

$$\text{ret}_t = \sigma_t \cdot \xi_t \tag{7}$$

$$\sigma_t = \sigma(v_t) \tag{8}$$

$$v_t = v(v_{t-1}, \text{ret}_{t-1}, \ldots). \tag{9}$$

A prominent example is the GARCH(1,1) process with $\sigma_t = \sqrt{v_t}$ and the conditional variance $v_t = \tilde{\omega} + \tilde{\beta}v_{t-1} + \tilde{\alpha}\text{ret}^2_{t-1}$ $(\tilde{\omega}, \tilde{\beta}, \tilde{\alpha} > 0)$. An example with v_t not restricted to positive values is the stochastic volatility model (SV) with $\sigma_t = \exp(v_t/2)$ and $v_t = \tilde{\gamma} + \tilde{\beta}v_{t-1} + \eta_t$ ($\tilde{\beta} > 0$, the random variable η_t may depend on ξ_t).

Starting from Eq. (6) (with $\bar{m}(t) \equiv m(t)$ for simplicity), by defining $\sigma_t = \sigma(L, \alpha m(t))$, $\sigma_0 = \sigma(L, 0)$, $v_t = (\sigma_t - \sigma_0)$ sign$(m(t))$ and using a Taylor expansion of $v_t(m(t-1) + \Delta m(t-1))$ and assuming $C = |\partial_m\sigma(L, \alpha m = 0)| \approx |\partial_m\sigma(L, \alpha m)|$ we obtain

$$\sigma_t = \sigma_0 + |v_t| \tag{10}$$

$$v_t = v_{t-1} + C\text{ret}_t. \tag{11}$$

This process is restricted to the subcritical regime. In the supercritical regime, correlations of the random variable ξ_t would have to be considered together with a jump of volatility. The latter has an analogue in regime switching models [9].

Let us continue by investigating the properties of the microscopic model using the macroscopic view. First we discuss the distribution of absolute returns. The single absolute return distributions $(|\Delta m| > 0)$ motivated by Eq. (6) have to be averaged:

$$g\left(|\Delta m|; \sigma(L, \bar{h}(t))\right) = \frac{2}{\sqrt{2\pi}\sigma}\exp\left(\frac{-\Delta m^2}{2\sigma^2}\right), \tag{12}$$

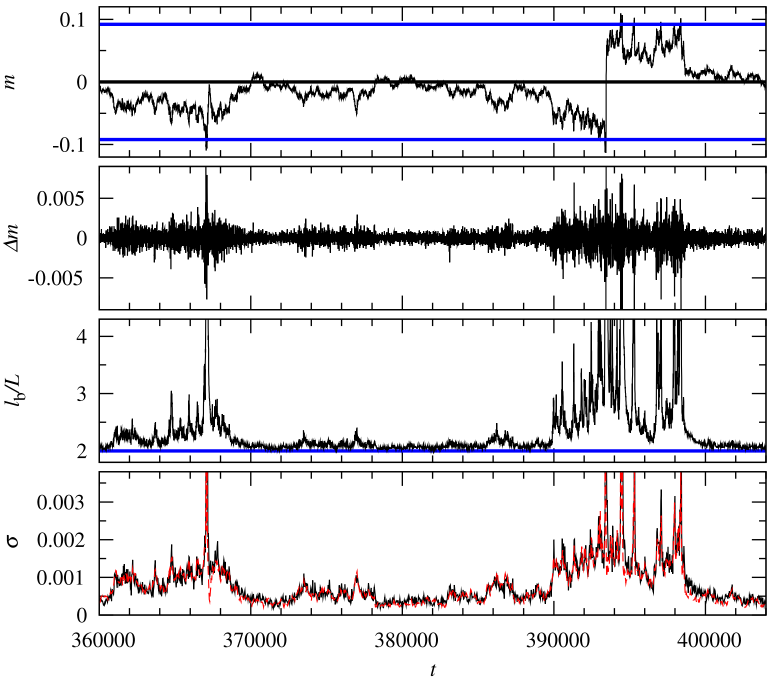

Fig. 3. From top to bottom: time series of magnetization m within the borders of $\pm m_{\rm crit}$; returns Δm; border length and minimal border length allowing for $m \approx 0$; standard deviation calculated as $\bar{\sigma}(t)$ (black) and as $\sigma(L, \alpha\bar{m}(t))$ (red dashed). The parameters are set to $L = 128$ and $\alpha = 20$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

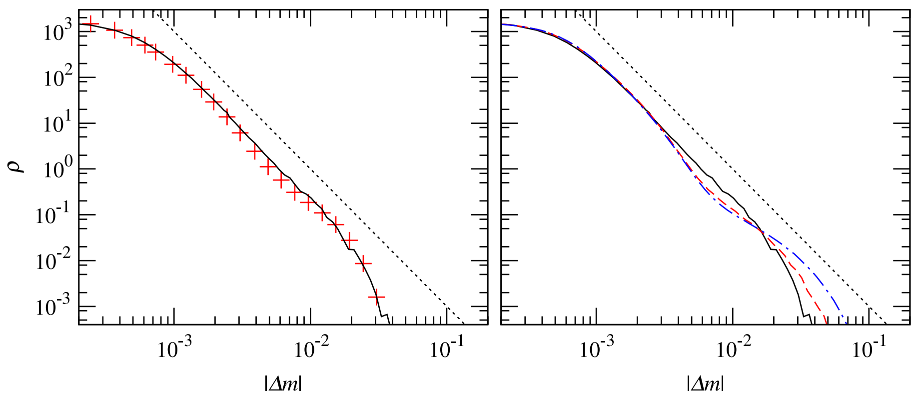

Fig. 4. Absolute return densities $\rho(|\Delta m|)$. Left: Comparison of the microscopic (solid line) and the macroscopic model (crosses) for $L = 128$, $\alpha = 20$. Right: $L = 128$, $\alpha = 20$ (solid line) together with rescaled densities for $L = 512$, $\alpha = 40$ (dashed) and $L = 2048$, $\alpha = 80$ (dash-dotted). Dotted lines show a power law with exponent -3. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\rho(|\Delta m|) = \frac{1}{T}\sum_{t=1}^{T} g\left(|\Delta m|; \sigma(L, \bar{h}(t))\right). \tag{13}$$

In Fig. 4 on the left the absolute return density for $L = 128$, $\alpha = 20$ is compared to the outcome of Eq. (13) with reasonable agreement. So the truncated power law distribution emerges due to the slowly but strongly varying $\sigma(t)$ as a superposition of Gaussian distributed returns with varying standard deviation for different times.

Finally we use the macroscopic view to discuss effects of the system size and feedback strength. This will enable us to adjust the feedback for increasing system size. There are two typical timescales in the system, the first due to the coarsening process during the quenching from supercritical to subcritical values h, the second due to the diffusive behavior in the subcritical region described by Eq. (6). For systems of increasing size, the ratio of both should be constant to guarantee similar time series. Additionally the total length of time series should be adjusted for the reason of convergence.

The coarsening process is similar to the GVM [30] with the consequence $l_b(t) \propto l_b(0) \cdot t^{-1/2}$. With $l_b(0) \propto L^2$ and $l_b(t_{\rm coarse}) \propto L$ we get $t_{\rm coarse} \propto L^2$. To estimate the typical subcritical timescale $t_{\rm sub}$ we argue that the partition of the random walk in magnetization broadens with time proportional to $\Delta m \cdot t^{1/2}$ and eventually will arrive at the borders $\pm h_{\rm crit}/\alpha$. With

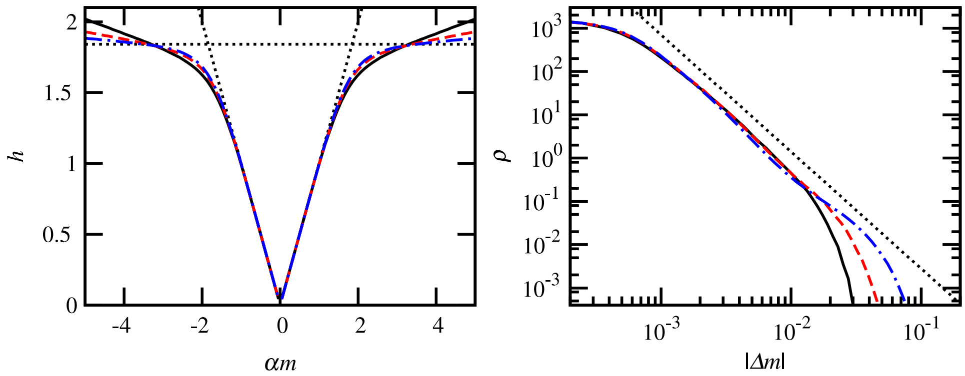

Fig. 5. Left: Modified feedback for the three cases of Fig. 4. Dotted lines show the linear feedback and h_{crit}. Right: Return densities as in Fig. 4 with the modified feedback. The dotted power law has exponent −2.7.

$\Delta m \propto L^{-3/2}$ in metastable states this defines a timescale $t_{sub} \propto (\alpha \Delta m)^{-2} \propto L^3/\alpha^2$. This is in fact found in the distribution of return-to-zero-times, which describes the bull and bear market durations. The cumulative distribution shows a power law with an exponent about 1/2, as can be found for uncorrelated random walks as well. The cutoff of this power law is close to $t_{cutoff} = L^3/\alpha^2$. Our estimate approaches the simulation within a factor of 2 for a wide range of parameters α and L from $t_{cutoff} = 32^3/10^2 = 328$ to $t_{cutoff} = 512^3/40^2 = 83\,886$.

With this knowledge we obtain the natural scaling of the feedback parameter as $\alpha \propto L^{1/2}$. On the right of Fig. 4 the case $L = 128$, $\alpha = 20$ (black line, $2 \cdot 10^6$ sweeps) is compared to $L = 512$, $\alpha = 40$ (red dashed, returns rescaled by two, $32 \cdot 10^6$ sweeps) and $L = 2048$, $\alpha = 80$ (blue dash-dotted, returns rescaled by four, $512 \cdot 10^6$ sweeps). For large returns a double-humped structure arises which is connected to the sharp phase transition (compare Fig. 2). The respective time series look indeed similar, if time is increased with $N = L^2$ and subsequently rescaled. Stronger feedbacks accelerate the dynamics, but the qualitative behavior of the time series changes fundamentally. For example the case $L = 512$, $\alpha = 140$ shows a dominant contribution of the coarsening process. Large consecutive jumps between $-h_{crit}$ and $+h_{crit}$ over long periods can be observed.

This observation motivates extensions of the spin model which we would like to add as a concluding remark. Allowing for non-linear feedback of the global magnetization allows to counteract this acceleration of large avalanches. In Fig. 5 the three cases of Fig. 4 are modified through a turn of the feedback shortly before the critical value:

$$h = \left[(\alpha|m|)^{-6} + \left(h_{crit} + 2|m| - 3.6/\sqrt{L} \right)^{-6} \right]^{-\frac{1}{6}}. \tag{14}$$

This feedback, here chosen as an interpolation of two limits of different linear slopes, conserves the timescales of the system, because the original linear feedback is maintained for the most part of subcritical excursions. The corresponding densities exhibit an extended scaling.

5. Outlook

In summary, by characterizing the system state of the spin market model [17] with the borderline length and by freezing the coupling to the global field, we found a phase transition and a dominant contribution of subcritical metastable striped states in the systems dynamics. We deduced a macroscopic equation for the price evolution and compared it to the microscopic model with reasonable agreement. We proceeded by investigating the model dynamics with the macroscopic point of view. With this method we found the mechanism leading to truncated power laws in the absolute returns, and we understood the timescales of the system originating from the coarsening process and from the subcritical diffusion periods. We found that the feedback strength should scale as $\alpha \propto L^{1/2}$ to balance the two typical timescales. Finally we used these findings to introduce an adaptive feedback that leads to extended scaling for larger systems whose return density tends to exhibit a substructure connected to the phase transition. In conclusion, while our main goal has been the microfoundation of macroscopic market equations, the deduced macro-equations also proved useful as a guiding principle for the construction and analysis of the microscopic model itself. It may serve as a prototype study for a similar approach to other agent based models.

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