Transient Dynamics Increasing Network Vulnerability to Cascading Failures

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We study cascading failures in networks using a dynamical flow model based on simple conservation and distribution laws. It is found that considering the flow dynamics may imply reduced network robustness compared to previous static overload failure models. This is due to the transient oscillations or overshooting in the loads, when the flow dynamics adjusts to the new (remaining) network structure. The robustness of networks showing cascading failures is generally given by a complex interplay between the network topology and flow dynamics.

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Societies rely on the stable operation and high performance of complex infrastructure networks, which are critical for their optimal functioning. Examples are electrical power grids, telecommunication networks, water, gas and oil distribution pipelines, or road, railway, and airline transportation networks. Their failure can have serious economic and social consequences, as various large-scale blackouts and other incidents all over the world have recently shown. It is therefore a key question how to better protect such critical systems against failures and random or deliberate attacks [1–3]. Issues of network robustness and vulnerability have not only been addressed by engineers [4,5], but also by the physics community [2,3,6–17]. In the initial studies of this kind [2,6–8,15], the primary concern was dedicated to what can be termed structural robustness: the study of different classes of network topologies and how they were affected by the removal of a finite number of links and/or nodes (e.g., how the average network diameter changed). It was concluded that the more heterogeneous a network is in terms of, e.g., degree distribution, the more robust it is to random failures, while, at the same time, it appears more vulnerable to deliberate attacks on highly connected nodes [7,15].

Later on, the concepts of network loads, capacities, and overload failures were introduced [9–14]. For networks supporting the flow of a physical quantity, the removal of a node or link will cause the flow to redistribute with the risk that some other nodes or links may be overloaded and failure prone. Hence, a triggering event can cause a whole sequence of failures due to overload, and may even threaten the global stability of the network. Such behavior has been termed cascading failures. A seminal work in this respect is the paper by Motter and Lai [9]. These authors defined the load of a node by its betweenness centrality [3,9]. Subsequent studies introduced alternative measures for the network loads [11] as well as more realistic redistribution mechanisms [11–14].

In all studies cited above, the redistribution of loads is treated in a time-independent or static way. We will refer to them collectively as static overload failure models. The load redistributions in such models are instantaneously and discontinuously switched to the stationary loads of the new (perturbed) network; i.e., the transient dynamical adjustment towards the new stationary loads of the perturbed network is neglected.

The aim of this Letter is to compare robustness estimates of complex networks against cascading failures where the dynamical flow properties are taken into account relative to those where they are not (static case). This work does not intend to target a specific system (or network); instead, we aim at being as generic as possible in the choice of dynamical model with the consequence that particular details and features of a specific system have to be neglected; i.e., we work with a minimal model as often favored in physics. Nevertheless, the conceptually simple dynamical phenomenological flow model that we propose incorporates flow conservation, network topology, as well as load redistribution features that are shared by real-life systems. On this background, it is expected (cf. Figure 1) that the model results will reflect some important properties of real-life systems.

For matters of illustration and to facilitate comparison with previous results [9–14], we have worked with topologies of power transmission networks. Although our model seems to capture stylized features of electrical networks (see Fig. 1), we stress that our goal is not a realistic representation of those, nor is our model restricted to such systems. Within the proposed model, we want to demonstrate that time-dependent adjustments can play a crucial role.

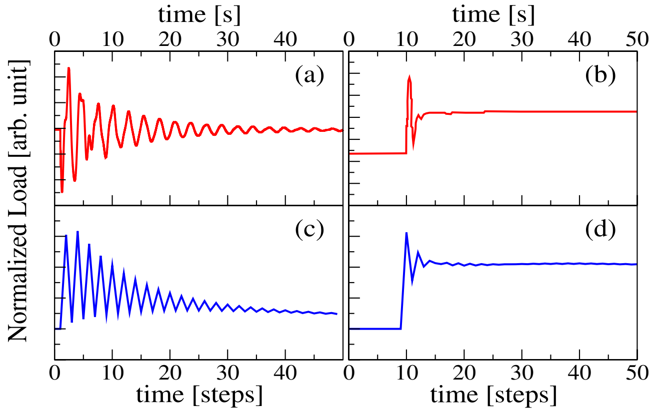

FIG. 1 (color online). Comparison of the time-dependent link loads after a triggering event (taking place at $t = 0$) as predicted by state-of-the-art power simulators [(a) [25] and (b) [26]], and the simple "flow-conserving" model described and used in the present Letter [(c) and (d)].

In the very tradition of physics, we use a simple flow model with few parameters, which however considers the network topology, flow conservation, and the distribution of loads over the neighboring links of a node [18,19]. We assume a network consisting of $\mathcal{N}$ nodes and represent it by a matrix $\mathbf{W}$, whose entries $W_{ij} \geq 0$ (with i, $j = 1, 2, \ldots, \mathcal{N}$) shall reflect the weight of the (directed) link from node j to i (with $W_{ij} = 0$ indicating no link present). The relative weights $T_{ij} = W_{ij}/w_i$ shall define the elements of the transfer matrix $\mathbf{T}$, where $w_j = \sum_{i=1}^{\mathcal{N}} W_{ij}$ is the total outgoing weight of node j [19]. These elements describe the distribution of the overall flow (per unit weight) $c_j(t)$ reaching node j at time t over the neighboring links i. When the flow is assumed to reach the neighboring nodes i at time step $t+1$, we obtain $c_i(t+1) = \sum_{j=1}^{\mathcal{N}} T_{ij}c_j(t) + j_i^{\pm}$ [18–20], where we have added possible source terms ($j_i^{\pm} > 0$) or sink terms ($j_i^{\pm} < 0$). In vectorial notation, the network flow equation reads

$$\boldsymbol{c}(t+1) = \boldsymbol{T}\boldsymbol{c}(t) + \boldsymbol{j}^{\pm}, \qquad (1)$$

resembling Kirchhoff's first law from circuit theory.

If $\boldsymbol{j}^{\pm} = 0$, the stationary solution to Eq. (1) is a constant vector with components $c_i^{(0)}(\infty) \sim 1/\sqrt{\mathcal{N}}$, while with a source term present ($\boldsymbol{j}^{\pm} \neq 0$), it can be expressed as $\boldsymbol{c}(\infty) = \boldsymbol{c}^{(0)}(\infty) + (\mathbf{1} - \boldsymbol{T})^{+}\boldsymbol{j}^{\pm}$ with $(\mathbf{1} - \boldsymbol{T})^{+}$ denoting the so-called generalized inverse [21] of the singular matrix $\mathbf{1} - \boldsymbol{T}$. Hence, the total directed current on link $j \longrightarrow i$ at time t becomes $C_{ij}(t) = W_{ij}c_j(t)$, from which also the (undirected) load $L_{ij}(t)$ of this link can be defined via $L_{ij}(t) = C_{ij}(t) + C_{ji}(t)$ [22]. Closed-form expressions for the flow dynamics at single nodes have been derived in Ref. [20]. These allow one to study the wavelike spreading and dissipation of perturbations in the network while propagating via neighboring links, second-next, etc. (see

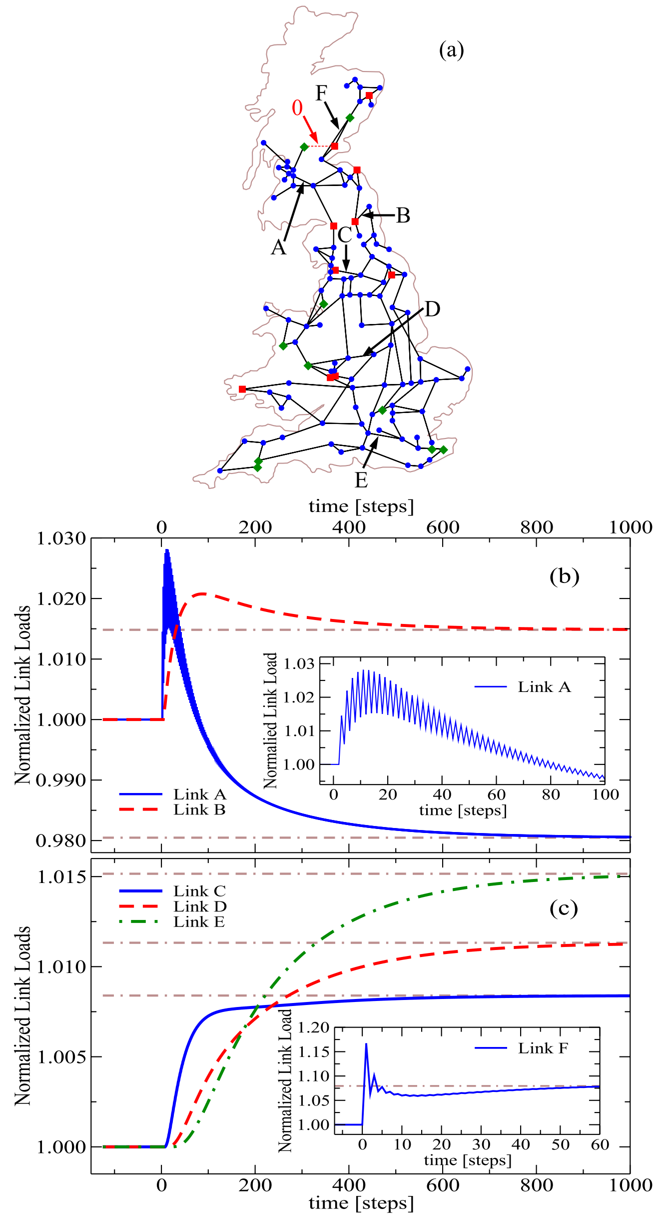

FIG. 2 (color online). (a) Illustration of the dynamics of our network flow model assuming the topology of the UK high-voltage power transmission grid (300–400 kV) consisting of 120 geographically correctly placed nodes (generators, utilities, and transmission stations) and 165 links (transmission lines). The network was treated as unweighted and undirected. In our simulations, twenty of the existing network nodes were chosen randomly to play the role of generator (source) and utility (sink) nodes ($|n_i^{\pm}/\mathcal{N}| = 2.5 \times 10^{-4}$), ten of each kind. In (a), the location of these nodes is indicated by squares and diamonds, respectively. At time $t = 0$, before which the network loads were in the stationary state [$L_{ij}(t < 0)$], the network was perturbed by removing a transmission line in Scotland [the dashed link marked by 0 in (a)]. The resulting normalized transient link loads, $L_{ij}(t)/L_{ij}(t < 0)$, are depicted in (b) and (c) for some selected links of the UK transmission grid, as indicated in (a). The horizontal dash-dotted lines correspond to the normalized stationary loads of the links.

Fig. 2). Such perturbations may result from the redistribution of flows after the failure of an overloaded link.

In the seminal work of Motter and Lai [9], failure of a node was based on the long-term overload; i.e., a node was assumed to fail whenever the stationary load in the perturbed network (considering previously broken nodes) exceeded the node capacity. The (node) capacities were defined as $1 + \alpha$ times the (stationary) loads of the original network with $\alpha \geq 0$ being a global tolerance factor (i.e., a relative excess capacity or safety margin). In other words, the evaluation of overloading was previously done after the system relaxed, without considering the time-history of how it got to this state (static overload failure models) [9–14].

In this Letter, we generalize this approach towards a dynamical overload failure model. Specifically, in our computer simulations, we assumed a link from node j to i to be overloaded (and to fail) whenever the time-dependent load $L_{ij}(t)$ exceeded the link capacity $\mathcal{C}_{ij}$ for at least a time period τ, the overload exposure time. The link capacities were defined analogously to Motter and Lai [9] as

$$\mathcal{C}_{ij} = (1 + \alpha)L_{ij}, \tag{2}$$

where L_{ij} denote the stationary loads of the original network.

In the following, we study the transient dynamical effects and overload situations that may occur before the stationary state is reached. While for $\tau = 0$, a failure results immediately after a first-time overload, $\tau > 0$ implies that the system will have to be overloaded for a certain time period in order to cause a failure. The static overload failure model corresponds to $\tau \to \infty$, or in practice, $\tau \gg \tau_0$, where τ_0 denotes the transient time of the system (the inverse of the smallest nonzero eigenvalue of $\boldsymbol{T}$). Therefore, the ratio $\chi = \tau/\tau_0$ can be used to interpolate between the static ($\chi \to \infty$) and (instantaneous) dynamical overload failure ($\chi = 0$) models. While the static overload failure model describes the upper limit of network robustness (the best case), the dynamic overload failure model with $\tau = 0$ gives the lower limit (the worst case) due to an overshooting flow dynamics (Fig. 2). Realistic cases are expected to lie between these two limiting cases, corresponding to a finite value of χ.

Apart from network robustness to overload failures, the value of χ also determines the dynamics of failure cascades. In the dynamic case with $\chi = 0$, close-by links are more likely to be overloaded and to fail than in the static case ($\chi \to \infty$). Therefore, in the dynamic scenario, one tends to have a pronounced "failure wave" sweeping over the network.

In order to further illustrate the difference between the static and dynamic cascading failure models, as well as getting a quantitative measure of the level of overestimation of robustness, we investigate one of the networks

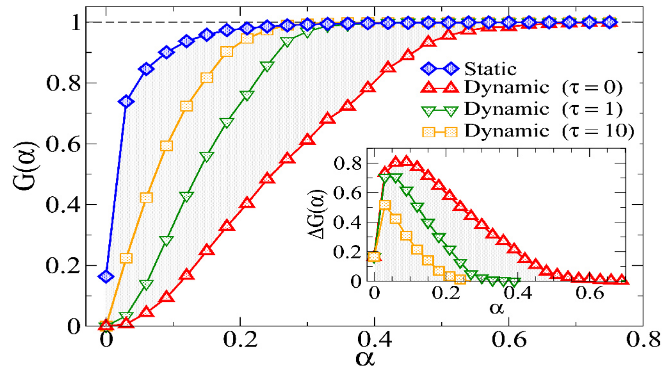

FIG. 3 (color online). The robustness of the Northwestern US power transmission grid [23], consisting of 4941 nodes, with an average node degree of 2.67 (cf. also Refs. [13,15,27]). The average fraction of links (or nodes), $G(\alpha)$, remaining in the giant component of this network (after cascading) is depicted as function of the tolerance parameter α, using the static and dynamic overload failure models described in the text. To obtain these results, the links were assigned weights, drawn from a uniform distribution on the interval [1, 10], and 200 generator and utility nodes of strength $|n_i^{\pm}/\mathcal{N}| = 10^{-8}$ were assigned randomly (100 of each type). The results were obtained by averaging over all possible triggering events (single link removals). The inset shows the difference, $\Delta G(\alpha)$, between the static and dynamic overload failure models.

already studied by Motter and Lai [9]—the Northwestern American power transmission network obtained from Ref. [23] (see also Refs. [15,23]). To evaluate the effect of an initial network perturbation and the following cascade (if any), we study the fraction of nodes and links, $G_N(\alpha)$ and $G_L(\alpha)$, respectively, remaining in the giant component of the network after potential cascading failures have ceased, which have been initiated by the random failure of a link [9]. Both quantities behave similarly [24]. They are displayed in Fig. 3 for the US power transmission network as functions of the tolerance parameter α. It has been checked and found that our static overload failure model well reproduces the general behavior previously reported in Ref. [9]. Namely, global cascading failure will occur under random attacks (or failures) mainly for heterogeneous networks.

According to Fig. 3, there is a pronounced difference between the static and (instantaneous) dynamic overload failure model, corresponding to upper and lower estimates to the network robustness. As is shown in the inset to Fig. 3, it can be as significant as 80%, and for more homogeneous link weights, we have found differences even higher than 95%. Only for quite significant tolerance factors ($\alpha \geq 50\%$), the discrepancy between the two estimates becomes insignificant. Moreover, it has also been found that the static model tends to be more sensitive to the location of sources (and sinks). Thus, our results show that the role of the dynamical process taking place on the network can be important when estimating the robustness of networks to failures and random attacks. It is not only the topology of the network that matters, but also the properties of the network dynamics as measured by $\chi = \tau/\tau_0$. The change in one or both of them will require a new robustness estimate.

In conclusion, we have simulated a simple network flow model considering, besides network topology, a flow-conserving dynamics and distribution of loads. Within this framework, we have studied the role of the transient dynamics of the redistribution of loads towards the steady state after the failure of network links. This transient dynamics is often characterized by overshootings and/or oscillations in the loads, which may result in characteristic "failure waves" spreading over the network. We have furthermore found that considering only the loads in the steady state (the static overload model) gives a best case estimate (upper limit) of the robustness. The worst case (lower limit) of robustness can be determined by the instantaneous dynamic overload failure model and may differ considerably.

Our simple dynamical approach provides additional insights into systems in which network topology is combined with flow, conservation, and distribution laws. These are potentially useful to understand, better design and protect critical infrastructures against failures. For instance, overloads related to high electrical currents cause (through overheating of wires) a slow spreading of failures as compared to the adjustment dynamics of the currents. This corresponds to $\chi \gg 1$. In contrast, within the validity limits of Ohm's law, one may also use our model to mimic effects of overloads related to overvoltages. In this case, we have $\chi \ll 1$, and link failures reflect the anticipatory disconnection of lines to prevent damages of the network and its components. Other examples, besides electrical power grids, are traffic systems, where overloaded streets cause unreasonably long travel times along links, which may be interpreted as effective link failures. The resulting choice of alternative routes corresponds to a rebalancing of loads and is expected to cause transient effects, with finite values of χ.

As the model allows for effective simulations, it could also be useful for close to real-time planning and optimization of network topologies and load sharing, particularly for large networks. Fully realistic state-of-the-art simulation tools for, say, electrical power grids that include network capacities, inductors, power generation, etc., are computationally expensive and therefore not so well suited for real-time simulation of large networks or their topological optimization. Hence, simpler models could quickly and efficiently give a useful overview that could serve as the starting point for more detailed off-line simulations using classical power network simulators.

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